# Options for Expressing Daily Loads in TMDLs

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Office of Wetlands, Oceans & Watersheds

June 22, 2007

**DRAFT**

# Appendix B: Identifying Daily Expressions for Non-daily Concentration-based TMDLs

Some TMDLs rely on establishing a concentration-based loading capacity, often equivalent to an applicable numeric water quality criterion. As with load-based TMDLs, if the established concentration-based TMDL is not on a daily time step, the TMDL should also include a daily expression representing the non-daily allocation. This appendix presents an approach for identifying a daily expression corresponding to the non-daily allocations developed in concentration-based TMDLs.

Numeric water quality standards or other water quality targets (representing narrative water quality criteria) have a duration component. For some criteria or targets, the duration is expressed as a daily average or *never to exceed* value. As an example, waters designated for support of semi-permanent, warm-water fish life in South Dakota must not exceed a daily maximum of 158 mg/L TSS. For concentration-based TMDLs established to meet these targets, the TMDL is already expressed on a daily basis. However, many water quality criteria or representative TMDL targets are based on longer time steps, including monthly or even annual averages. Figure 25 illustrates an example TMDL developed to attain the water quality criterion of an annual average concentration of 25 mg/L TSS. For concentration-based TMDLs set equivalent to longer-term targets, the TMDLs should also include a daily expression.

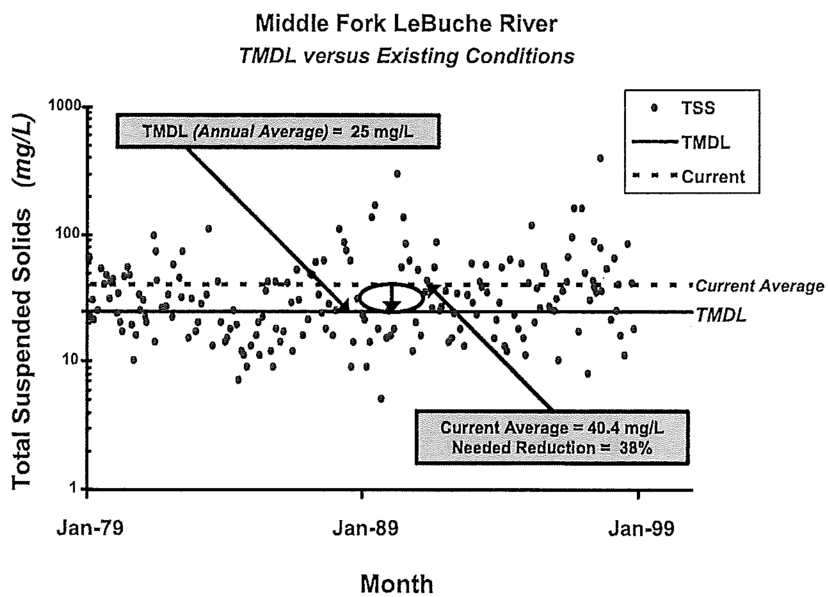


**Figure 25. Example of a concentration-based TMDL.**

The daily expression representing the non-daily concentration-based TMDL should account for variability occurring in the system. Water quality and quantity vary over time in terms of volumes discharged and constituent concentrations. Variations occur because of a number of factors, including changes in weather conditions, precipitation, seasonality, and source inputs. Figure 25 shows how concentrations vary for a parameter when water quality data are plotted against time.

Understanding the variability associated with water quality conditions is a key part of evaluating an impaired waterbody. Water quality at a location over time can be described using common descriptive statistics, such as the monthly or annual average concentration, the standard deviation, and the coefficient of variation. The coefficient of variation is a statistical measure of the relative variability of a dataset and is defined as the ratio of the standard deviation to the mean. Another way to describe water quality patterns is by constructing a *frequency-concentration* plot of the data. Figure 26, for example, depicts the Middle Fork LeBuche TMDL with a frequency-concentration plot of data that reflects attainment of water quality standards.

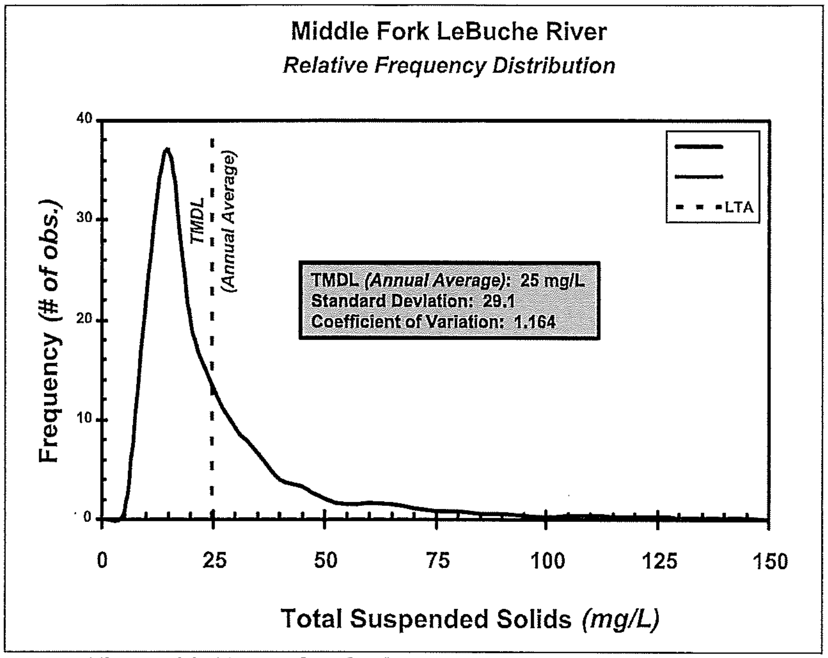


**Figure 26. Example of a frequency-concentration plot.**

On the basis of the frequency-concentration curve's shape, data can be described in terms of a particular type of statistical distribution. Choices often include a normal distribution (bell-shaped), lognormal distribution (positively skewed), or other variations on the lognormal distribution. EPA's *Technical Support Document for Water Quality-Based Toxics Control* (USEPA 1991) uses lognormal distributions to determine maximum daily and monthly average effluent limits, based on achieving a long-term average (LTA) target and an understanding of variability.

The TSD provides a statistical framework to identify a target maximum daily concentration corresponding to an LTA and based on a coefficient of variation and the assumption of a lognormal distribution. The equation for determining the maximum daily limit (MDL) is as follows (USEPA 1991):

$$MDL = LTA \times e^{[z\sigma - 0.5\sigma^2]},$$

where

*MDL* = Maximum daily limit
*LTA* = Long-term average (in the same units as the MDL)
*Z* = z-score associated with target recurrence interval
$\sigma^2 = \ln(CV^2 + 1)$
*CV* = Coefficient of variation

Details regarding the mathematics used to derive this equation are described in USEPA (1991).

The z-score is sometimes called the *standard score* for normal distributions because it provides a useful way to compare sets of data with different means and standard deviations. The z-score for an item (or a particular recurrence interval) indicates how far and in what direction that item deviates from its distribution's mean (expressed in units of its distribution's standard deviation). For instance, a z-score of +1.0 indicates that item (or recurrence interval) is one standard deviation in the positive direction from the mean. Z-scores are published in basic statistical reference tables and are often included as a spreadsheet function (e.g., NORMSINV(y) in Microsoft Excel).

Using this relationship, the TSD includes a table of *LTA to MDL* multipliers for several recurrence interval/coefficient of variation combinations (USEPA 1991). Table 18 provides a summary of these multiplier values for several averaging periods used in TMDL development (e.g., 30-day, 60-day ...365-day). These averaging periods are also expressed as a recurrence interval to identify the appropriate z-score for use in the equation. For example, the daily maximum of a 30-day averaging period equates to a 96.8 percent recurrence interval (e.g., [30/31]% or [k/k+1]% where k is the number of averaging period days) with a corresponding z-score of 1.849. If the coefficient of variation for a parameter is 1.0, the multiplier to convert the LTA to an MDL is 3.30 (**Note***:* key boxes for this combination are shaded in Table 18).

**Table 18. Multipliers used to convert an LTA to MDL**

| Averaging period (days) | Recurrence interval | Z-score | Coefficient of variation | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 |
| 30 | 96.8% | 1.849 | 1.41 | 1.89 | 2.39 | 2.87 | 3.30 | 3.67 | 3.99 | 4.26 | 4.48 |
| 60 | 98.4% | 2.135 | 1.50 | 2.11 | 2.80 | 3.50 | 4.18 | 4.81 | 5.37 | 5.87 | 6.32 |
| 90 | 98.9% | 2.291 | 1.54 | 2.24 | 3.05 | 3.91 | 4.76 | 5.57 | 6.32 | 7.00 | 7.62 |
| 120 | 99.2% | 2.397 | 1.58 | 2.34 | 3.24 | 4.21 | 5.20 | 6.16 | 7.06 | 7.89 | 8.66 |
| 180 | 99.4% | 2.541 | 1.62 | 2.47 | 3.51 | 4.66 | 5.87 | 7.06 | 8.20 | 9.29 | 10.3 |
| 210 | 99.5% | 2.594 | 1.64 | 2.52 | 3.61 | 4.84 | 6.13 | 7.42 | 8.67 | 9.86 | 11.0 |
| 365 | 99.7% | 2.778 | 1.70 | 2.71 | 4.00 | 5.51 | 7.15 | 8.83 | 10.5 | 12.13 | 13.7 |

Figure 27 graphically illustrates a *log probability plot* of the EPA equation using data that reflect conditions associated with attainment of the water quality standards. The x-axis is expressed as the z-score of a normal probability distribution; the y-axis displays concentrations on a logarithmic scale. A probability plot is one method that can be used to check the assumption of lognormality. If the data follow the pattern of a lognormal distribution, they will fall approximately along a straight line, as shown in Figure 27.

Figure 27 also shows translation of the recurrence interval for an annual averaging period (e.g., 365 days) to the corresponding maximum daily concentration limit. The following calculations demonstrate identification of the MDL on the basis of the corresponding LTA:

$$MDL = LTA \times e^{[z\sigma - 0.5\sigma^2]},$$

where

$LTA$ = 25 mg/L

$z$ = 2.778 (based on recurrence interval of 99.7%)

$CV$ = 1.164

$$\sigma^2 = \ln(CV^2 + 1) = \ln(1.164^2 + 1) = 0.857$$

Therefore,

$$MDL = 25\frac{mg}{L} \times e^{[2.778 \times 0.926 - 0.5 \times 0.857]} = 25\frac{mg}{L} \times 8.533 = 213.3\frac{mg}{L}.$$

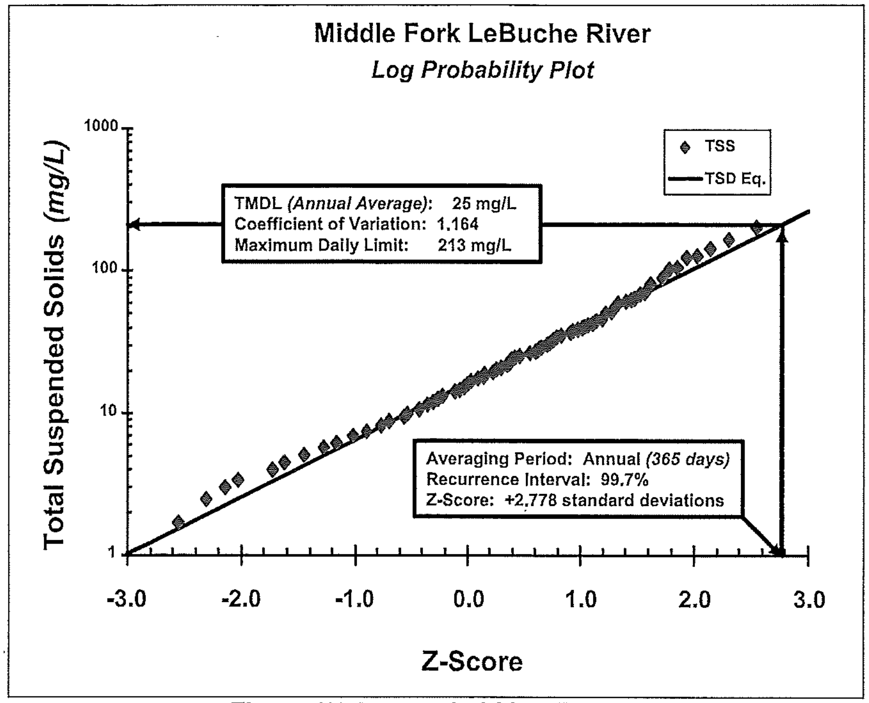


Figure 27. Log probability display.